The 1D-Greenberg-Hastings cellular automaton

We describe excitable media discretely in time and space by using the alphabet $A = \{0, 1, 2, \ldots, e, \ldots \}$ composed of the rest state 0, the excited states $E = \{1, 2, \ldots \}$ and the refractory states $R = \{s + 1, \ldots, e + r \}$. Throughout, we assume that $\epsilon < 1$. The local transition rule $T: \mathbb{A}^2 \to \mathbb{A}$ is given by

**Total dynamics:**

$$T((i,j)) = \begin{cases} 
0 & \epsilon \leq 1 < e + 1, \\
(0,0) & \epsilon = 1, \\
(0,1) & \epsilon < 1.
\end{cases}$$

**State graph:**

$$\begin{array}{ccc}
& L & \\
\downarrow & \downarrow & \\
R & \varnothing & E
\end{array}$$

**State transition graph:**

$$\begin{array}{ccc}
& E & \\
\downarrow & \downarrow & \\
R & L & \varnothing
\end{array}$$

Inherent state loop:

$$\begin{align*}
\frac{\partial}{\partial t} \phi(y) & = \frac{\partial}{\partial x} \phi(y), \\
\phi(0) & = \phi(L), \\
\phi(L) & = \phi(0).
\end{align*}$$

In the hypothetical case $\epsilon = 1$, the topological entropy $h(Y)$ can be computed directly using (11L16). Moreover, $h(Y) = \lim_{n \to \infty} \frac{1}{n} \log |\mathcal{T}_n|$, where $\mathcal{T}_n$ is the set of all $n$-periodic configurations. The following theorem states that the topological entropy of the 1D-GHCA is always equal to zero.

**Theorem 1:** Topological entropy of 1D-GHCA (1D-GHCA).

$$h(Y) = \lim_{n \to \infty} \frac{1}{n} \log |\mathcal{T}_n| = 0.$$