Universität Bremen

ALTA
Institute for Algebra, Geometry, Topology and their Applications

One-day workshop on CONVEX GEOMETRY

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On L_p Brunn-Minkowski inequalities for general absolutely continuous measures

Lidia Gordo Malagón

(joint work with J. Yepes Nicolás)

The L_p version (for $p \geqslant 1$) of the Brunn-Minkowski inequality proven by Firey in the 60's for two convex bodies containing the origin, and recently extended to non-empty compact sets $K, L \subset \mathbb{R}^n$ by Lutwak, Yang and Zhang, asserts that the volume is a (p/n)-concave functional, namely, that for all $\lambda \in (0,1)$

$$\mathrm{vol}\big((1-\lambda)\cdot K +_p \lambda\cdot L\big)^{p/n} \geqslant (1-\lambda)\mathrm{vol}(K)^{p/n} + \lambda \mathrm{vol}(L)^{p/n}.$$

Moreover, equality, for some $\lambda \in (0,1)$ and $p > 1$, holds if and only if K and L are dilatates and contain the origin.

In this talk, we will collect various Brunn-Minkowski type inequalities, jointly with their respective equality cases, for a general class of absolutely continuous measures with radially decreasing densities when dealing with the p-sum of the sets involved. In those results, particular families of sets must be considered, so that one may aspire to get the maximum degree of concavity, namely p/n, as it happens with the volume functional.

On characterizations of (dual) mixed volumes

María de los Ángeles Hernández Cifre

(joint work with D. Alonso)

Given m convex bodies in the n-dimensional Euclidean space $\mathbb{R}^n$, there are $N_{n,m} = \binom{n+m-1}{n}$ mixed volumes associated to them. In 1960 Shephard posed the question whether the known inequalities relating the mixed volumes (Aleksandrov-Fenchel and some determinantal inequalities) were enough in order to characterize them, in the following sense: given $N_{n,m}$ non-negative real numbers satisfying the inequalities, do there exist n convex bodies whose mixed volumes are the given numbers? We have also considered the corresponding Shephard problem in the dual Brunn-Minkowski theory, i.e., to look for necessary and sufficient conditions for a set of positive real numbers to be the dual mixed volumes of star bodies in $\mathbb{R}^n$.

In this talk we will present these appealing problems on (dual) mixed volumes as well as their similarities and significant differences, and we will show the known results, recent advances and open questions regarding them.

Canal Classes and Cheeger Sets

Nico Lombardi

(joint work with Christian Richter and Eugenia Saorín Gómez)

Giannopoulos, Hartzoulaki and Paouris asked whether the best ratio between volume and surface area of convex bodies sharing a given orthogonal projection onto a fixed hyperplane is attained in the limit by a cylinder over the given projection. Although the answer to this question is known to be negative in general, in this talk we present a characterization of when a positive answer holds in dimension 3, using the Cheeger set of the common projection. A partial characterization is also given in higher dimensions. Additionally, we present close connections between this question and other types of geometric inequalities.

Jesús Yepes Nicolás

(joint work with A. Zvavitch)

In this talk, we will discuss various functional and geometric forms of “reverse” Brunn-Minkowski type inequalities, in both their dual form and their complemented version. We will study these inequalities in the setting of different absolutely continuous measures on $\mathbb{R}^n$ with radially decreasing densities, by paying special attention to the cases of the volume (the n-dimensional Lebesgue measure) and the standard Gaussian measure.