Statistical methods, WS 13/14

Problems

1. (5 points) Let $X_1, ..., X_n$ be i.i.d random variables from a distribution F. Construct a test for checking the null hypothesis that the distribution function is indeed F under the alternative that it is not F such that a power of the test tends to 1 with $n \to \infty$ with any alternative.

2. (5 points) Let $X_1, ..., X_n$ be i.i.d random variables from a uniform distribution $\mathcal{U}[\theta, 2\theta]$, $\theta \in I\!R$. In order to estimate θ consider an estimator $T = T(\boldsymbol{X}) = \alpha X_{(1)} + \beta X_{(n)}$, $\alpha, \beta \geq 0$, $X_{(i)}$ is the ith order statistics. Find α and β such that T is efficient and unbiased.

3. (10 points) Let (X_1, X_2, X_3) be i.i.d random variables from a distribution $\mathcal{N}(0, \theta^2)$. Find an efficient estimator for $f(\theta) = P(X_1 \leq x_0)$ with some fixed $x_0 \in I\!R$.

4. (5 points) A function $x(t) = \beta_2 t^2 + \beta_1 t + \beta_0$ is measured at points t_i $(i = 1, ..., n)$:

$$X_i = \beta_2 t_i^2 + \beta_1 t_i + \beta_0 + \epsilon_i, \quad E\epsilon_i = 0, \, \mathrm{Var}(\epsilon_i) = \sigma^2,$$

σ^2 is known. In order to estimate an integral $I = \int_a^b x(t)dt$ consider an estimator $\widehat{I} = \int_a^b \widehat{x}(t)dt$, where $\widehat{x}(t)$ is a least squares estimator of $x(t)$. Is $\widehat{I}$ an unbiased estimator of I? Find $\mathrm{Var}(\widehat{I})$.

5. (10 points) Let X be a r.v. from a normal distribution $\mathcal{N}(\mu, 1)$, $\mu \in I\!R$. Given one observation of the r.v. X in order to estimate μ consider a function $f(X)$ from a class $\mathcal{F} = \{f : I\!R \to I\!R \big|$ f is continuous and $I\!E f(X)^2 < \infty\}$. Prove that the minimum of $\sup_{\mu \in I\!R} E(f(X) - \mu)^2$ over all $f \in \mathcal{F}$ is achieved with $f(x) = x$.

These problems are optional. They can bring you extra points. Solutions should be submitted before the first seminar on Monday, 6 January 2014.