Exercises for the lecture on

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Statistical Methods

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Sheet 3

Solution are due on Monday, November 4th, 2013, 3:15pm.
Every completely and correctly solved exercise gives 4 points.

Exercises

9. **Properties of estimators.**
   - (a) Show that convergence of the quadratic risk of an estimator $\hat{\varrho}_n(X)$ of $\varrho(\vartheta)$ towards zero for increasing sample size $n \to \infty$ implies consistency of $\hat{\varrho}_n(X)$.
   - (b) Does the convergence under (a) also imply strong consistency of $\hat{\varrho}_n(X)$?

10. **Support estimation**. Let $(X_k)_{k \in \mathbb{N}}$ denote an iid. sequence of real-valued random variables, where $X_1 \sim \text{UNI}[\vartheta, \vartheta + 1]$. Consider the following two sequences of estimators of $\vartheta \in \Theta = \mathbb{R}$.

$$\hat{\vartheta}_n^{(1)}(X_1, \ldots, X_n) = \bar{X}_n - 1/2 = n^{-1} \sum_{k=1}^{n} X_k - 1/2,$$
$$\hat{\vartheta}_n^{(2)}(X_1, \ldots, X_n) = \min\{X_1, \ldots, X_n\}.$$

   - (a) Compute the bias of $\hat{\vartheta}_n^{(j)}(X_1, \ldots, X_n)$ for $j = 1, 2$.
   - (b) Compute the variance and the quadratic risk of $\hat{\vartheta}_n^{(j)}(X_1, \ldots, X_n)$ for $j = 1, 2$.
   - (c) Determine if $\hat{\vartheta}_n^{(j)}(X_1, \ldots, X_n)$ is consistent, for $j = 1, 2$.
   - (d) Find a deterministic transformation of $\hat{\vartheta}_n^{(2)}(X_1, \ldots, X_n)$ which estimates ϑ unbiasedly.

11. **Programming exercise.** Consider again Example 1.22 (one-sided binomial test).
   - (a) Verify (by curve sketching) that $F(p, k)$ is a monotonously decreasing function in $p \in \Theta = [0, 1]$ for any fixed $k \in \Omega$.
   - (b) How large is the probability to detect the alternative, if the true "success probability" is exactly equal to the empirically observed relative frequency of the target event (tumor-caused death)? To answer this question, compute the power of the non-randomized test φ in the point $p_1 = 5/13 \in \Theta_1$ by making use of statistics software.
   - (c) Which minimum sample size n would have been necessary such that the power of φ in $p_1 = 5/13$ (under the constraint $\mathbb{E}_{1/5}[\varphi] \leq 5\%$) is larger than 0.9? Compute this minimum required sample size with statistics software. What does a normal approximation yield with respect to this problem?

12. **Multiple Select.** Which of the following statements are true and which are false? Please give reasons for your respective decisions (one short sentence each is sufficient).
   1. The size of a Bayes test (see, for instance, Example 1.10) for a simple null hypothesis under 0-1 loss depends on the choice of the prior distribution.

2. If the p-value for a given test problem exceeds 0.5, then the Bayesian posterior distribution assigns more mass to the null hypothesis than to the alternative, no matter the prior.
3. For a fixed sample size n, there exist only finitely many significance levels α such that the size of a non-randomized binomial test for a simple null hypothesis $\{p_0\}$ equals α.
4. If a test φ is of Neyman-Pearson type, then the corresponding p-value for a given realization $x \in \Omega$ is an antitone transformation of the value $T(x)$ of the underlying test statistic.