Exercises for the lecture on

Statistical Methods

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Sheet 4

Solution are due on Monday, November 11th, 2013, 3:15pm.
Every completely and correctly solved exercise gives 4 points.

Exercises

13. **Distribution theory.**
    (a) Prove Lemma 1.32.
    (b) Prove Lemma 1.34.
    (c) Prove Theorem 1.35.

14. **Testing a Gaussian variance.** Whenever a test item is weighed on the scales several times, (slightly) different measurement values are obtained. We assume that the weighings can be modeled well by stochastically independent, identically $\mathcal{N}(\mu, \sigma^2)$-distributed random variables, where μ (in g) represents the unknown true mass of the test item (no systematic bias). Furthermore, we assume (for ease of argumentation) that the positive constant σ^2 reflects the precision of the scales and is not influenced by any other experimental factors. The manufacturer of the scales provides the information $\sigma^2 = \sigma_0^2 = 5 \times 10^{-5}$ (in g^2). This indication of quality shall be tested at significance level $\alpha = 5\%$. Among 16 test weighings of a randomly chosen test item, we observe an empirical variance of $s^2 = 6.1 \times 10^{-5}$ (in g^2).
    (a) Does our measurement result imply significant (at level $\alpha = 5\%$) evidence against the manufacturer's indication of quality (one-sided test)?
    (b) Derive the power function of the one-sided test that has been used in (a). How large is the probability not to detect the alternative if the true variance is indeed exactly equal to the empirically observed one, i. e., if $\sigma^2 = s^2 = 6.1 \times 10^{-5}$?
    (c) What minimum sample size is necessary such that the probability under (b) becomes smaller than $\beta = 0.2$?

15. **Programming exercise.** Two different chemical compounds A and B for colouring fabric are investigated. The fabric company is especially interested in compounds that are insensitive to fading of colours by sunlight. To assess the respective properties of A and B, ten randomly chosen swatches are cut into halves, the two halves are coloured (by a standardized procedure) with the different compounds and are then exposed to direct sunlight (by a standardized procedure). After that, the colour intensity of each piece is measured on the wool scale (small measurement values correspond to stronger fading of colours). The measurement results are listed in the following table.

Swatch	1	2	3	4	5	6	7	8	9	10
Compound A	7.2	4.3	5.8	6.5	4.9	6.8	6.3	7.0	6.5	6.2
Compound B	5.1	4.1	5.5	4.1	5.0	5.1	5.3	7.3	4.8	5.8

(a) Based on these data, compute the p-value for the null hypothesis that there is no difference in the mean sunlight sensitivity of the two compounds by making use of statistics software. In this, assume that the distribution of the differences of the swatch-specific wool scale values (A - B) can be modeled well by a normal distribution. The variance of this normal distribution is unknown.

(b) On the basis of the available data, do you think that the normal distribution assumption made in (a) is a reasonable model?

16. **Multiple Select.** Which of the following statements are true and which are false? Please give reasons for your respective decisions (one short sentence each is sufficient).

    1. The median and the mean of a χ^2_ν-distribution coincide if $\nu > 2$.
    2. For every $\nu \in \mathbb{N}$, the 95%-quantile of the t_ν-distribution is larger than the 95%-quantile of the standard normal distribution.
    3. The measure $\mathbb{P}^*$ for computing the p-value for the one-sided test problem 1) on the handout is the standard normal distribution.
    4. Fisher's F-distribution with first degree of freedom equal to one and second degree of freedom equal to $\nu \in \mathbb{N}$ is the distribution of the square of a t_ν-distributed random variable.