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Quasisymmetric functions and Eulerian enumeration

We describe some links between enumeration of chains in Eulerian posets and questions about representations of certain quasisymmetric functions. The commutative *peak algebra* Π of Stembridge is generated by quasisymmetric functions arising from enriched P-partitions. The noncommutative algebra $A_{\mathcal{E}}$ consists of all chain-enumeration functionals $\sum \alpha_S f_S$ on Eulerian posets. Both have Hilbert series given by the Fibonacci numbers. Bergeron, Mykytiuk, Sottile and van Willigenburg have shown that, with natural coproducts, Π and $A_{\mathcal{E}}$ are dual Hopf algebras. As a consequence, for a rank $n+1$ Eulerian poset P, the quasisymmetric function $F(P) = \sum_{S \subset [n]} f_S(P)\, M_S$ is always an element of Π. We study this pairing explicitly and show that the coefficients of the **cd**-index for Eulerian posets, as elements of $A_{\mathcal{E}}$, form a dual basis to that given by the weight-enumerators Θ_S of enriched P!-partitions of labelled chains. Thus Eulerian posets P that have nonnegative **cd**-indices are precisely those that are Θ-positive. These include all face posets of convex polytopes and are conjectured to include all Gorenstein* posets.

This is joint work with Samuel K. Hsiao and Stephanie van Willigenburg

http://www.math.cornell.edu/ billera/papers/peaks.ps