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The matroid Grassmannian is homotopy equivalent to the real Grassmannian

The set of rank k oriented matroids on an n-element set is partially ordered by specialization, or weak maps. Besides its inherent combinatorial interest, the order complex of this poset, known as the matroid Grassmannian, has been much studied as a potential gateway through which combinatorial techniques might be brought to bear on topology. Indeed, it has already been used by Gelfand and MacPherson as the essential basis for their combinatorial formula for the rational Pontrjagin classes.

We show that the matroid Grassmannian is homotopy equivalent to the Grassmannian of k-planes in a rank n real vector space. This suggests that matroids may have further use in the study of characteristic classes, namely in the computation of integral Pontrjagin classes, as well as elsewhere in differential topology.