P-kernels, IC bases, and Kazhdan-Lusztig polynomials

In 1992 Stanley introduced, for any locally finite poset P, the concept of a P-kernel. This concept includes as special cases several interesting objects, such as Kazhdan-Lusztig polynomials and the local intersection cohomology Poincaré polynomials of toric varieties. In 1994 Du introduced the concept of an IC basis. Examples of IC bases are the Kazhdan-Lusztig bases of Hecke algebras of Coxeter groups and of q-Schur algebras, as well as the canonical bases of quantized enveloping algebras and of quantum linear groups.

In this talk we show that any P-kernel corresponds to an IC-basis and, under some mild hypotheses, conversely. We also characterize, given a Coxeter group W partially ordered by Bruhat order, among all W-kernels the one corresponding to the Kazhdan-Lusztig basis of the Hecke algebra of W. Finally, we show that this W-kernel factorizes as a product of other W-kernels, that these provide a solution to the Yang-Baxter equations for W, and we compute explicitly the IC bases corresponding to them.