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Lower central series and free resolutions of arrangements.

A well-known formula of Falk and Randell computes the ranks of the lower central series quotients of the fundamental group of the complement of a supersolvable hyperplane arrangement in terms of the Poincaré polynomial. We seek to show that there exist other, large classes of arrangements for which a modified LCS formula holds, with the Poincaré polynomial replaced by another (combinatorially determined) polynomial.

Determining the LCS ranks, ϕ_k , is equivalent to determining the Betti numbers, $\text{Ext}_A^i(\mathbb{Q}, \mathbb{Q})_i$, of the linear strand of the free resolution of the residue field, viewed as a module over the Orlik-Solomon algebra, A . We use the change of rings spectral sequence to relate these Betti numbers to the resolution of A over the exterior algebra E , recovering a formula of Falk for ϕ_3 , and obtaining a new formula for ϕ_4 .

In the case of graphic arrangements, we make a precise conjecture, expressing the LCS ranks in terms of the clique polynomial of the graph. In the case of 3-arrangements with minimal linear strand, we describe the free resolution of A over E , and make another conjecture, expressing the LCS ranks in terms of the Möbius function of the intersection lattice. We verify both conjectures in low ranks.

This is joint work with Henry K. Schenck (Texas A&M University).