A B_n -analogue of the chessboard complex

The chessboard complex is the simplicial complex of rook placements on an $m \times n$ chessboard. It arises in various contexts in the literature and has a number of interesting topological properties. The chessboard complex, a B_n analogue of it and a wreath product generalization of it were introduced and put in the framework of a Tits coset complex by Garst in 1979. We extend results on the chessboard complex dealing with homotopy, representations of the symmetric group on rational homology and torsion in integral homology, to the B_n -analogue and the wreath product generalization. A new technique in topological combinatorics is developed in the process.