

ARRANGEMENTS OF HYPERPLANES
Algebra, Combinatorics, Geometry and Topology

Centro Stefano Franscini, Ascona, Switzerland, May 16 – 20, 2005

ABSTRACTS

Takuro Abe (Kyoto University):

Title: Freeness of arrangements and an endomorphism sheaf on the intersection lattice

Abstract: Sheaves and its cohomology groups on intersection lattices are studied by Yuzvinsky to solve some freeness problems of arrangements. From the viewpoint of algebraic geometry, we relate it with the sheaf cohomology on projective spaces, and we also characterize the freeness of arrangements by defining and using an endomorphism sheaf of logarithmic vector fields.

Silvano Baggio (Università di Bologna):

Title: Two applications of sheaves on simplicial complexes

Abstract: We present two results which have been found out using in two different situations a similar method (that develop techniques introduced by Baclawski and Yuzvinsky), involving the study of sheaves on partially ordered sets (in our case: simplicial complexes). The first result is a computation, under certain assumptions, of the terms E_2 of a spectral sequence (due to Merkurjev) which connects equivariant and ordinary K-theory of smooth toric varieties. See arXiv:math.AG/0410378. The second one is a short proof of Reisner's Theorem characterizing which Stanley-Reisner rings of simplicial complexes are Cohen-Macaulay.

Daniel C. Cohen (Louisiana State University):

Title: Gauss-Manin connections for arrangements

Abstract: We study the Gauss-Manin connection for the moduli space of arrangements of fixed combinatorial type in the cohomology of the complement with coefficients in a rank one local system. For nonresonant local systems, we solve the eigenvalue problem for the endomorphisms arising in the one-form associated to the Gauss-Manin connection. This is joint work with Peter Orlik.

Péter Csorba (ETH Zurich):

Title: The universality and other properties of homomorphism complexes

Abstract: For two graphs G and H Lovasz introduced a cell complex $Hom(G, H)$. It gives nontrivial topological lower bounds for the chromatic number. I will show that they are universal, thereby answering the question of Matousek and Ziegler about Sarkaria's topological bound. We will characterize the graphs G for which

$Hom(G, K_n)$ are manifolds, and, in this way, obtain a new and rich class of graph coloring manifolds. The most interesting example is $G = C_5$: it plays an important role in Lovasz' conjecture as well, proven recently by Babson and Kozlov. We will show that $Hom(C_5, K_4)$ is homeomorphic to the 3-dimensional projective space. This agrees with the conjecture stating that $Hom(C_5, K_n)$ is homeomorphic to the Stiefel-Manifold $V_{n-1,2}$. This is partly joint work with Frank Lutz.

Sonja Čukić (KTH Stockholm):

Title: Topology of graph coloring complexes

Abstract: For any two graphs G and H , Lovász defined a cell complex $Hom(G, H)$ with the idea that topological and algebraic invariants of these complexes should provide non-trivial lower bounds for chromatic numbers of graphs.

One instance of Hom complexes is the one when a complete graph is the second argument. In this case, $Hom(G, K_n)$ can be understood as a complex of graph colorings of G , since its vertex set is exactly the set of all allowed colorings of G which use at most n colors.

We will present a proof of a conjecture by Babson and Kozlov:
If the maximal valency of G is d , then $Hom(G, K_n)$ is at least $(n - d - 2)$ -connected.
 This is joint work with D. Kozlov.

Emanuele Delucchi (ETH Zurich):

Title: Subdivision of complexes of k -trees

Abstract: The complex of k -trees is the abstract simplicial complex whose faces correspond to combinatorial types of rooted trees with $(n-1)k+1$ labelled leaves and all outdegrees congruent to 1 modulo k (and larger than $k+1$). Face relations among these trees are given by contraction of internal edges. This complex has very similar combinatorial and homological properties as the poset of partitions of $(n-1)k+1$ with block sizes equal to 1 modulo k . Using the theory of combinatorial nested sets, Feichtner proved that the order complex of the partition lattice of n elements is a subdivision of the complex of 1-trees. In this talk the proof of the analogous statement for general k will be presented. The result is reached by an ad-hoc generalization of concepts of the theory of nested set complexes to non-semilattices.

Ehud de Shalit (Hebrew University):

Title: p -adic hyperplane arrangements and harmonic analysis on buildings

Abstract: Given a (possibly infinite) hyperplane arrangement in a vector space V over a p -adic field, one may reduce it modulo p with respect to various integral structures. To the resulting arrangements over the finite residue field we can attach a system of Orlik-Solomon algebras. This system can be interpolated to give a local system of graded algebras on the Bruhat-Tits building of $PGL(V)$. We shall explain our work on this local system and the applications to the cohomology of p -adically uniformized varieties. Recent results of G. Alon and E. Grosse-Klonne will also be explained.

Alexandru Dimca (University of Nice):

Title: Geometry of polynomial mappings transversal to infinity

Abstract: In this talk, based on a joint work with A. Libgober, I will show that a polynomial transversal to infinity behaves in many respects just like a homogeneous polynomial. In particular, I will explain the relations to old results by Kulikov and Kulikov (concerning mixed Hodge structures on Alexander invariants of plane curves) and to recent results by Maxim (who was the first to study this natural class of polynomials).

Igor Dolgachev (University of Michigan):

Title: Sheaves of logarithmic differentials

Abstract: I will review some properties of the sheaves of logarithmic differentials attached to hyperplane arrangements. Among them are problems of sheaf stability, Torelli type theorems, and a GIT-stability related to the action of the group of projective transformations on the space of arrangements.

Alexander Engström (ETH Zurich):

Title: Complexes of directed forests

Abstract: To a directed graph one can associate the complex of all subgraphs which are directed forests. We will show that a large class of subcomplexes are shellable and determine their homotopy type.

Michael Falk (Northern Arizona University):

Title: Multi-nets, resonant weights, and pencils of curves

Abstract: Let $\mathcal{A}$ be a projective line arrangement, with complement X . A resonant pair is a pair (ω, η) of non-proportional logarithmic 1-forms on X satisfying $\omega \wedge \eta = 0$. We say $\mathcal{A}$ supports (ω, η) if ω or η has poles along every line of $\mathcal{A}$. A multi-net is a multi-arrangement of lines in the projective plane, with relatively prime multiplicities, partitioned into classes such that (i) each class contains d lines, counting multiplicities, and (ii) each inter-class intersection point is contained in the same number of lines from each class, counting multiplicities. Any line arrangement supporting a resonant pair of weights is the underlying arrangement of some weak multi-net. At the level of matroids, every weak multi-net is a degeneration (i.e., weak image) of a net.

A multi-net satisfies an additional general position requirement. Every multi-net arises from a pencil of degree d curves whose (not necessarily reduced) singular elements include all the lines of $\mathcal{A}$ counting multiplicities. As a corollary, generic linear combinations of ω and η have non-isolated zero loci. Under some additional numerical conditions, the pencil yields a fibration of X by punctured Riemann surfaces, consequently X is aspherical.

This is joint work with Sergey Yuzvinsky.

Kwai-Man Fan (National Chung Cheng University):

Title: Some notes on the fundamental group of the complement of an arrangement of complex hyperplanes

Abstract: First, we study the splitting as a direct product of the fundamental group of the complement of an arrangement of complex hyperplanes under some combinatorial conditions. Second, we apply a certain method to generate some examples of arrangements of complex lines such that their intersection lattices have finitely many projectively inequivalent realizations in the complex projective plane.

Giovanni Gaiffi (Scuola Normale Superiore, Pisa):

Title: Models of arrangements and polytopes

Abstract: We will discuss some combinatorial aspects of the theory of real and complex De Concini - Procesi models of subspace arrangements. In particular we will focus on the case of Coxeter arrangements, describing a class of polytopes and cohomological properties.

Yukihito Kawahara (Tokyo Metropolitan University):

Title: On the construction of arrangements with the non-vanishing twisted cohomology

Abstract: Under the generic situation, the cohomology with coefficients in the local system on complements of hypersurfaces vanishes except in the highest dimension. In the case of arrangements of hyperplanes, many examples were founded. By using decomposable relations arising from Latin hypercubes, I will explain some construction of arrangements with non-vanishing cohomology of Orlik-Solomon algebras. Furthermore, I would like to generalize their examples to hypersurfaces.

Toshitake Kohno (University of Tokyo):

Title: Resonance at infinity and the space of conformal blocks

Abstract: The purpose of this talk is to give a description of the space of conformal blocks in terms of the homology of local systems over the complement of discriminantal arrangements. A basis of the space of conformal blocks is given by hypergeometric integrals over certain twisted cycles. Here the difficulty arises since we deal with resonant local systems. By means of compactification of the complement of discriminantal arrangements and the associated Leray spectral sequence we clarify a relation between regularizable cycles and the space of conformal blocks.

Paulo Lima-Filho (Texas A&M University):

Title: On the holonomy Lie algebra of graphic arrangements

Abstract: In joint work with Hal Schenck, we use the holonomy Lie algebra to obtain an explicit combinatorial formula for the ranks of the lower central series quotients

of the fundamental group of the complement of a graphic arrangement. This formula generalizes Kohno's result for braid arrangements, and provides the first instance of a lower central series formula for a large class of arrangements which are not decomposable or fiber-type.

Stefan Papadima (Romanian academy):

Title: Equivariant chain complexes and twisted homology

Abstract: The equivariant chain complex of the universal cover, associated to a Morse-theoretic minimal cell structure, X , on an arrangement complement, turns out to be independent of X , thus defining a new arrangement invariant. A similar result holds for Alexander covers of arbitrary minimal complexes. We point out a connection between the relative minimality problem, and the homology of the Milnor fiber.

Mario Salvetti (Università di Pisa):

Title: On the homology of complexes associated to poset diagrams and applications

Abstract: We introduce a class of algebraic complexes, associated to certain "diagrams" over posets, and study some homology of them. Applications to the homology of Artin groups and Milnor fibers are found.

Frank-Olaf Schreyer (Universität des Saarlandes):

Title: Computing the higher direct image complex of a coherent sheaf

Abstract: The higher direct image complex of a coherent sheaf (or finite complex of coherent sheaves) under a projective morphism is a fundamental construction that can be defined via a Čech complex or an injective resolution, both inherently infinite constructions. Using exterior algebras and relative versions of theorems of Beilinson and Bernstein-Gel'fand-Gel'fand, we give an alternate description in finite terms.

Using this description we can give explicit descriptions of the loci in the base spaces of flat families of sheaves in which some cohomological conditions are satisfied—for example, the loci where vector bundles on projective space split in a certain way, or the loci where a projective morphism has higher dimensional fibers.

This is joint work with David Eisenbud.

Bernd Sturmfels (UC Berkeley):

Title: Combinatorial secant varieties

Abstract: Secant varieties of arrangements of coordinate subspaces lead to interesting operations on simplicial complexes. We present on-going joint work with Seth Sullivant on the algebra, topology and combinatorics of these objects, including a new approach to determinantal and Pfaffian ideals and a surprising connection to the strong perfect graph theorem.

Alexandru I. Suciu (Northeastern University):

Title: Free partially commutative groups and Lie algebras

Abstract: We show how to express various invariants of a free partially commutative group or Lie algebra in terms of the associated graph.

Alexander Varchenko (University of North Carolina):

Title: Critical points of master functions, multiple orthogonal polynomials, and a counterexample to the Gaudin Bethe Ansatz conjecture

Abstract: I will discuss relations between multiple orthogonal polynomials and critical points of master functions associated with the Gaudin model.

Max Wakefield (University of Oregon):

Title: Complete intersection apolar algebras

Abstract: We study the following question: when is the differential operator ring with constant coefficients modulo the annihilator ideal of a polynomial a complete intersection. In the case of arrangements, for this quotient ring to be a complete intersection, skew-symmetry of the polynomial is more important than the arrangements combinatorics or its freeness. Then we examine the generalization to an arbitrary polynomial.

Masahiko Yoshinaga (Kyoto University):

Title: The Lefschetz theorem and hyperplane arrangements

Abstract: I will talk about how top cells attach to a generic hyperplane section in Lefschetz hyperplane section theorem for the complement of a complexified real arrangement.

I also give a combinatorial description of the twisted cellular chain complex for minimal CW-decomposition of the complement.