Steven H. Weintraub
Lehigh University

Spreads of non-singular pairs in symplectic vector spaces

Let V be a vector space of dimension $2n$ over a field $\mathbf{F}$ equipped with a non-singular symplectic form $<,>$.

Def. A **non-singular pair** (or **ns-pair**) in V is $\Delta = \{\delta, \delta^\perp\}$ consisting of a pair of n-dimensional subspaces, mutually orthogonal, whose direct sum is V, and such that the restriction of $<,>$ to each of them is non-singular.

Def. A **spread of non-singular pairs** or **nsp-spread** is a set of ns-pairs $\sigma = \{\Delta_i\}$ whose non-zero elements partition $V - \{0\}$ (regarding Δ as the union of the elements in δ and $\delta^\perp$).

Theorem. *For any even n, any field $\mathbf{F}$ that is either a finite field of odd characteristic or an algebraic number field, and any vector space V of dimension $2n$ over $\mathbf{F}$, as above, there exist nsp-spreads in V.*

$\Sigma = \{\text{nsp-spreads } \sigma \text{ in } V\}$ is an interesting algebraic/combinatorial object, which we discuss. This theorem is new except in case $n = 2$ and $\mathbf{F} = \mathbf{F}_3$, in which case (as Coble knew in 1908) the 27 elements of Σ correspond to the 27 lines on the non-singular cubic surface.