(updated)

Workshop on Arrangements

Forschungsinstitut für Mathematik, ETH Zürich
December 6–8, 2000

Abstracts

Daniel Cohen (Louisiana State University, Baton Rouge)

Lie algebras associated to fiber-type arrangements

Recent results of Fadell-Husseini and Cohen-Gitler reveal an interesting relationship between the Lie algebras arising (i) from the lower central series of the Artin pure braid group, and (ii) from the loop space homology of certain classical configuration spaces. We discuss how these results extend naturally to fiber-type hyperplane arrangements and certain higher dimensional analogues.

Michael Falk (Northern Arizona University, Flagstaff)

Arrangements and matroids determined by their points and lines

We recall the notion of formality of arrangements, and proceed to discuss various combinatorial versions of formality and their implications for the topological and algebraic structure of arrangements. In particular, we outline recent work on quadratic algebras and line-closed matroids.

Giovanni Felder (ETH Zürich)

From local to global deformation quantization of Poisson manifolds

We give a construction of a deformation quantization of the algebra of functions on a Poisson manifold, based on Kontsevich's local formula. The deformed algebra of functions is realized as the algebra of horizontal sections of a bundle of algebras with a flat connection. The construction is based on a geometric-algebraic interpretation, in terms of connections and curvature, of four special cases of the Kontsevich formality theorem on $\mathbb{R}^d$.

In this talk, I will review the content of the formality theorem, which is based on Stokes' theorem on a compactification of the configuration space of n points on the upper half plane. I will then show how this result can be applied to the contruction of a deformation of the algebra of functions on a Poisson manifold.

This talk is based on joint work with A.S. Cattaneo and L. Tomassini.

Giovanni Gaiffi (Scuola Normale Superiore, Pisa)

Real models of arrangements and conical stratifications

Let us consider a central subspace and half-space arrangement $\mathcal{A}$ in an euclidean vector space V and let $\mathcal{M}(\mathcal{A})$ be its complement. We construct some compactifications for the C^∞ manifold $\mathcal{M}(\mathcal{A})/\mathbb{R}^+$. They turn out to be C^∞ manifolds with corners equipped with a nice combinatorial description of the boundary. This generalizes a construction described by Kontsevich in "Deformation quantization of Poisson manifolds, I." (q-alg/9709040). Then we extend the construction to more general objects, i.e., stratified real manifolds whose stratification locally "looks like" the one induced by a subspace and half-space arrangement.

Richard Randell (University of Iowa, Iowa City)

Cell structures on arrangement complements and Milnor fibers

We use Morse theoretic results of Le and Hamm to show that the Milnor fiber associated to an arrangement has a canonical cell structure compatible with the monodromy. This cell structure passes down to the complement of the associated affine arrangement and thence to the central arrangement. For the arrangement complements the structure is minimal (the number of k-cells equals the k-th betti number.) Following work of Papadima-Suciu this has implications for the homotopy theory of these spaces. This talk will explain this structure and explore the implications for homology of the Milnor fiber and the homotopy theory of the complements. Some description of attaching maps for these cells will be given.

Mario Salvetti (University of Pisa)

Combinatorial methods for the cohomology of Coxeter groups

Alexander Suciu (Northeastern University, Boston)

Fundamental groups of arrangements

Fundamental groups of complements of arrangements are difficult to study, yet enough light has been shed on their structure, that once seemingly intractable problems can now be attacked in earnest. A unifying framework for the study of arrangement groups is provided by their characteristic and resonance varieties. Those varieties, in turn, are connected in subtle ways to the combinatorics of the arrangement.

In this talk, I will discuss several aspects of fundamental groups of arrangements, such as homological and enumerative questions regarding their finite-index subgroups. I will focus on arrangements of affine lines in the complex plane, but will also consider arrangements of transverse planes in real 4-space.

HIROAKI TERAO (Tokyo Metropolitan University)
Multiderivations of Coxeter arrangements

Let V be an ℓ-dimensional Euclidean space. Let $G \subset O(V)$ be a finite irreducible orthogonal reflection group. Let $\mathcal{A}$ be the corresponding Coxeter arrangement. Let S be the algebra of polynomial functions on V. For $H \in \mathcal{A}$ choose $\alpha_H \in V^*$ such that $H = \ker(\alpha_H)$. For each nonnegative integer m, define the derivation module $\mathsf{D}^{(m)}(\mathcal{A}) = \{\theta \in \mathrm{Der}_S \mid \theta(\alpha_H) \in S\alpha_H^m\}$. The module is known to be a free S-module of rank ℓ by K. Saito (1975) for $m = 1$ and L. Solomon-H. Terao (1998) for $m = 2$. The main result of this paper is that this is the case for all m. Moreover we explicitly construct a basis for $\mathsf{D}^{(m)}(\mathcal{A})$. Their degrees are all equal to $mh/2$ (when m is even) or are equal to $((m-1)h/2) + m_i (1 \leq i \leq \ell)$ (when m is odd). Here $m_1 \leq \cdots \leq m_\ell$ are the exponents of G and $h = m_\ell + 1$ is the Coxeter number. The construction heavily uses the primitive derivation D which plays a central role in the theory of flat generators by K. Saito (or equivalently the Frobenius manifold structure for the orbit space of G.) Some new results concerning the primitive derivation D are obtained in the course of proof of the main result. Also relations to the "exponents" for the extended Shi arrangements, which are all equal to kh, and to the "exponents" for the extended Catalan arrangements, which are $kh + m_i (1 \leq i \leq \ell)$, will be discussed.

SERGEY YUZVINSKY (University of Oregon, Eugene)
Free arrangements and sheaves on projective spaces

GÜNTER M. ZIEGLER (Technische Universität Berlin)
Fat-lattice polytopes: some constructions
from reflection arrangements, rigidity, rationality

05/12/00